

### Exercises for Chapter 3

3.1 The cutting of small, square, metal plates (used to make square nuts) is done using a shearing process. The variables in the process include shearing pressure, shearing blade width, and the temperature of the metal to be cut. The response is the crispness of the cut, rated on a scale of 1 to 10. The measured (reported) crispness is an average crispness of 10 sheared plates. A process engineer decided to study the system to find the settings of the variables that would maximize crispness. The data are listed below.

Run Order	X <sub>1</sub> = Pressure - = 200 + = 400	X <sub>2</sub> = Width - = 0.2" + = 1.0"	X <sub>3</sub> = Temp. - = 300 °F + = 500 °F	Crispness
7, 4	-	-	-	3.1 4.2
1, 14	+	-	-	8.4 8.8
5, 12	-	+	-	3.8 4.6
10, 15	+	+	-	8.9 9.1
2, 11	-	-	+	3.2 4.3
3, 16	+	-	+	6.8 7.3
6, 13	-	+	+	8.0 8.4
8, 9	+	+	+	9.4 9.9

- Plot the response versus run and comment on what you see.
- Calculate the mean and variance,  $s^2$ , of the replicate responses for each run, and calculate the pooled standard deviation  $s_p$ .
- Make a plot of cell standard deviations versus cell means to see if the equal variance assumption is justified.
- Calculate the main effects, two-way interaction effects and the three-way interaction effects.
- Make boxplots of the response at each level of the three factors similar to Figure 3.20, and comment on what you see.
- Calculate the standard error of an effect,  $s_E$ , and determine which effects and interactions are significant at the 95% confidence level.
- Graph any significant interactions and write a sentence or two interpreting each significant effect or interaction.
- What conditions would maximize crispness?

3.2 An experiment is to be performed to determine the effects of Fill Pressure (20psi or 60psi), Barrel Temperature(180° or 200°) and Mold Temperature(180° or 200°) upon the percent shrinkage of injection molded parts.

- a) Set up the list of experiments for a  $2^3$  factorial in the standard order
- b) Choose a list of random run orders for the experiments.

3.3 Connors, Anderson, Perkins and Cedeno performed an experiment to determine the effects of Spindle Speed (- = 960RPM, + = 1800RPM), Feed Rate (- = 7.2IPM, + = 16IPM) and Depth of Cut (- = 0.005 in., + = 0.030in.) upon the surface finish of milled aluminum. The data is listed below; smaller values indicate a smoother finish.

$X_1 = SS$	$X_2 = FR$	$X_3 = DC$	Surface Finish
-	-	-	35 42 28
+	-	-	76 124 124
-	+	-	41 18 38
+	+	-	60 65 68
-	-	+	33 34 34
+	-	+	137 144 167
-	+	+	28 28 26
+	+	+	64 52 66

- a) Calculate the mean and variance,  $s^2$ , of the replicate responses for each run, and calculate the pooled standard deviation,  $s_p$ .
- b) Make a plot of cell standard deviations versus cell means to see if the equal variance assumption is justified.
- c) Calculate the main effects, two-way interaction effects, and the three-way interaction effect.
- d) Make boxplots of the response at each level of the three factors similar to Figure 3.20, and comment on what you see.
- e) Calculate the standard error of an effect,  $s_E$ , and determine which effects and interactions are significant at the 95% confidence level.
- f) Graph any significant interactions and write a sentence or two interpreting each significant effect or interaction.

3.4 The following data (with coded values for factors) were taken from a study dealing with a solar water heating system.<sup>1</sup> The factors were  $X_1$ , the total daily insolation,  $X_2$ , the tank storage capacity,  $X_3$ , the water mass flowrate through the absorber, and  $X_4$ , the intermittency of the input from solar radiation. The responses were  $Y_1$ , the collection efficiency, and  $Y_2$ , the energy delivery efficiency. A computer model was developed from theory, but was too complicated for ready appreciation. Therefore, the model was used to predict the responses at the sixteen conditions defined below according to a  $2^4$  design. This would allow for a simpler linear approximation model to be determined.

Run	$X_1$	$X_2$	$X_3$	$X_4$	$Y_1$	$Y_2$
1	-	-	-	-	43.5	82.0
2	+	-	-	-	41.3	82.0
3	-	+	-	-	44.9	82.1
4	+	+	-	-	43.0	82.2
5	-	-	+	-	35.0	61.7
6	+	-	+	-	37.5	66.0
7	-	+	+	-	39.7	67.7
8	+	+	+	-	39.9	68.6
9	-	-	-	+	51.3	83.7
10	+	-	-	+	50.2	86.3
11	-	+	-	+	52.4	84.1
12	+	+	-	+	51.9	89.8
13	-	-	+	+	38.4	100.0
14	+	-	+	+	39.2	100.0
15	-	+	+	+	41.3	100.0
16	+	+	+	+	41.6	100.0

- Calculate the effects for each response and construct a Normal or half-Normal plot of the effects to determine which effects and interactions appear to be significant.
- Make a Pareto Diagram of the absolute effects to determine which effects and interactions appear to be significant
- For each response, separately construct boxplots at each level of the four factors similar to Figure 3.20 and comment on what you see.
- Graph any significant interactions and write a sentence interpreting each significant main effect and/or interaction.

---

<sup>1</sup>Close, D. J. "A Design Approach for Solar Processes," Solar Energy, Vol. 11, p 112 (1967).

3.5 Lorenc, Jones, McKee, and Runyan performed a  $2^3$  experiment to determine the effects of machine settings on Boy 15s injection molding machine upon the weight of injection molded parts. The machine settings or factors they studied were Holding Time (- = 10 seconds, + = 20 seconds), Holding Pressure (- = 4 psi, + = 28 psi) and Injection Speed (- = 4, + = 80). The data is listed below.

$X_1=HT$	$X_2=HP$	$X_3 = IS$	Part Weight
-	-	-	13.62 13.60 13.59 13.64
+	-	-	13.61 13.67 13.60 13.61
-	+	-	13.63 13.72 13.74 13.70
+	+	-	13.71 13.72 13.70 13.61
-	-	+	13.81 13.80 13.82 13.81
+	-	+	13.78 13.79 13.79 13.76
-	+	+	14.24 14.32 14.32 14.26
+	+	+	14.24 14.11 14.24 14.56

- Calculate the mean and variance,  $s^2$ , of the replicate responses for each run, and calculate the pooled standard deviation  $s_p$ .
- Make a plot of cell standard deviations versus cell means to see if the equal variance assumption is justified.
- Calculate the main effects, two-way interaction effects and the three-way interaction effects.
- Make boxplots of the response at each level of the three factors similar to Figure 3.20, and comment on what you see.
- Calculate the standard error of an effect,  $s_E$ , and determine which effects and interactions are significant at the 95% confidence level.
- Graph any significant interactions and write a sentence or two interpreting each significant effect or interaction.

3.6 Huber, Hendricks, Nichol, and Peterson performed a  $2^3$  experiment to determine the effects of the CO concentration (- = 0.1, + = 0.2), H<sub>2</sub> concentration (- = 0.2, + = 0.4) and volumetric flowrate (- = 100 cc/min., + = 200 cc/min.) upon the output of a gas chromatograph used to measure the concentration of H<sub>2</sub>. The data is listed below.

X <sub>1</sub> =CO	X <sub>2</sub> =H <sub>2</sub>	X <sub>3</sub> =Flowrate	Peak Area × 10 <sup>-5</sup>
-	-	-	3.082 2.880 4.944
+	-	-	3.954 3.081 2.837
-	+	-	6.612 5.150 5.006
+	+	-	5.013 5.406 4.965
-	-	+	3.249 3.153 3.005
+	-	+	2.971 2.975 3.049
-	+	+	5.595 5.343 5.436
+	+	+	5.388 5.368 5.494

- Calculate the mean and variance,  $s^2$ , of the replicate responses for each run, and calculate the pooled standard deviation  $s_p$ .
- Make a plot of cell standard deviations versus cell means to see if the equal variance assumption is justified.
- Calculate the main effects, two-way interaction effects, and the three-way interaction effects.
- Make boxplots of the response at each level of the three factors similar to Figure 3.20, and comment on what you see.
- Calculate the standard error of an effect,  $s_E$ , and determine which effects and interactions are significant at the 95% confidence level.
- Graph any significant interactions and write a sentence or two interpreting each significant effect or interaction.

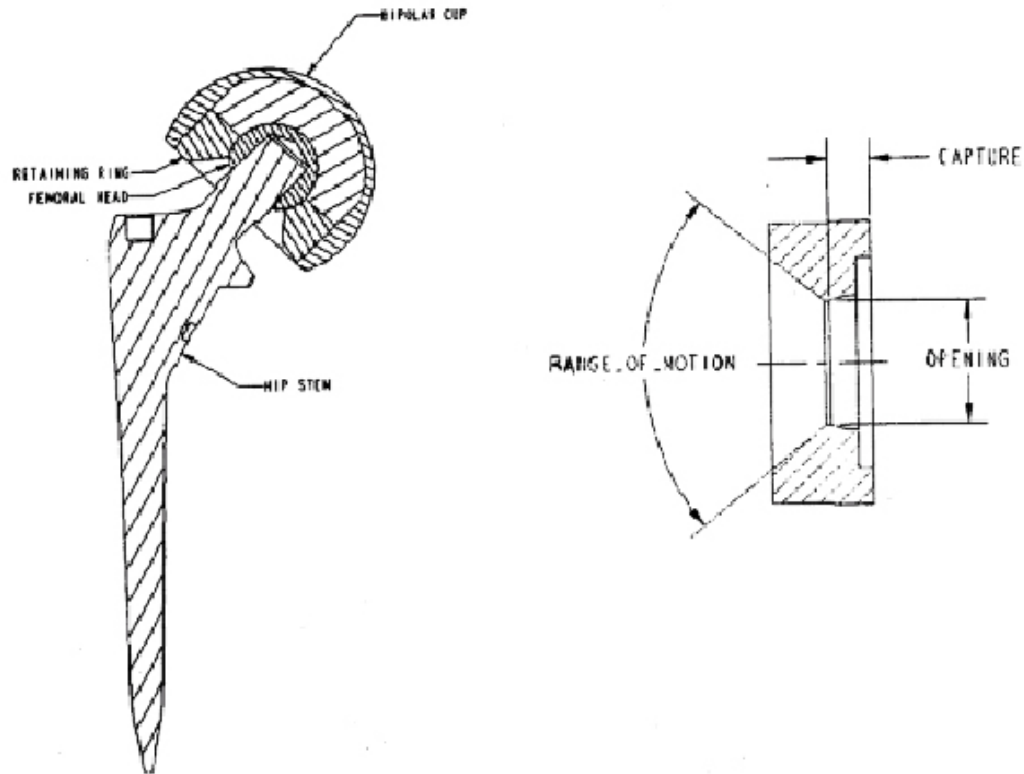


3.7 A quality engineer in a chemical production facility was assigned the task of maximizing the yield of a chemical reaction taking place in a continuous stirred tank reactor (CSTR). The four variables she decided to examine first were temperature, concentration of catalyst, agitator speed, and reactor pressure. A full factorial design was used, and the data are listed below.

Run	X <sub>1</sub> = Temp. - = 80 °C + = 100 °C	X <sub>2</sub> = Cat. - = 4 g/l + = 5 g/l	X <sub>3</sub> = Agit. - = 400 rpm + = 500 rpm	X <sub>4</sub> = Press. - = 700 psi + = 1000 psi	Yield
1	-	-	-	-	79
2	+	-	-	-	81
3	-	+	-	-	82
4	+	+	-	-	84
5	-	-	+	-	85
6	+	-	+	-	84
7	-	+	+	-	85
8	+	+	+	-	86
9	-	-	-	+	87
10	+	-	-	+	88
11	-	+	-	+	89
12	+	+	-	+	91
13	-	-	+	+	93
14	+	-	+	+	94
15	-	+	+	+	95
16	+	+	+	+	97

- Calculate the main effects and all interactions regardless of order.
- Estimate the standard error of an effect,  $s_E$ , from the high order interactions. Then determine which effects and interactions are significant at the 95% confidence level.
- Make a normal plot of the effects, and a Pareto Diagram of the absolute effects and judge graphically which effects and interactions are significant. How does your result there compare to what you found in b)?
- Graph any significant interactions and write a sentence or two interpreting each significant effect or interaction.
- What conditions would maximize yield?

3.8 Wagner, Stockett, Swensen, and Wright studied the factors that affect the force to dissociate the femoral head from the bipolar cup in a hip replacement socket. In order to perform hip replacement, doctors need to cut through a membrane surrounding the hip joint. This membrane holds the hip tightly in place, and does not recover after surgery. Therefore a hip replacement is more susceptible to dislocation than a healthy hip joint. When a hip is dislocated, the doctor tries to move the hip back into its correct position without surgery. This sometimes results in a dissociation of the bipolar cup and necessitates surgery to replace the cup.



Four factors were studied to find their effects upon the force necessary to dissociate the bipolar cup. These factors are listed below:

<u>Factor</u>	(-) <u>Low Level</u>	(+) <u>High Level</u>
Capture Height (CH)	3mm	8mm
Retaining Ring Opening (OP)	Narrow	Wide
Femoral Head Offset (OFF)	+0mm	+5mm
Ring Size (RS)	42mm	58mm

The following table lists the experimental settings and results obtained by making and testing 16 prototype bipolar cups. The objective was to maximize the force to dissociate while retaining at least 49 degree range of motion.

Run Order	CH	OP	OFF	RS	Force to dissociate	Range of Motion
10	3mm	Narrow	0mm	42mm	58.5	52.5
2	8mm	Narrow	0mm	42mm	88	52.5
4	3mm	Wide	0mm	42mm	39	58.69
16	8mm	Wide	0mm	42mm	46.5	58.6
9	3mm	Narrow	5mm	42mm	49.5	39.5
5	8mm	Narrow	5mm	42mm	66.5	39.5
8	3mm	Wide	5mm	42mm	32	47.2
3	8mm	Wide	5mm	42mm	46.5	47.2
7	3mm	Narrow	0mm	58mm	49.5	54.1
11	8mm	Narrow	0mm	58mm	102	54.1
1	3mm	Wide	0mm	58mm	32.5	59.5
12	8mm	Wide	0mm	58mm	58.5	59.5
14	3mm	Narrow	5mm	58mm	36	41.1
15	8mm	Narrow	5mm	58mm	81	41.1
6	3mm	Wide	5mm	58mm	29	48.3
13	8mm	Wide	5mm	58mm	56.5	48.3

- Plot the response versus run and comment on what you see.
- Calculate the effects for each response, and construct a Normal or Half-Normal plot of the effects to determine which effects and interactions appear to be significant.
- For each response, separately construct boxplots at each level of the four factors similar to Figure 3.20 and comment on what you see.
- Graph any significant interactions and write a sentence interpreting each significant main effect and/or interaction.